Exam. Code : 211001
Subject Code : 5472

M.Sc. Mathematics I[st] Semester

REAL ANALYSIS–I

Paper–MATH-551

Time Allowed—3 Hours] [Maximum Marks—100

Note :– Attempt **two** questions from each Unit. All questions carry **10** marks each.

UNIT–I

1. What is an open sphere in a metric space ? What are the open spheres in discrete metric space ? Prove that collection of all arbitrary union of open spheres is closed under finite intersections. Also give an example of two different metrics on a set for which the collection of all arbitrary unions of open spheres is same.

2. Prove that there cannot be any surjection from the set of integers to the set of all subsets of integers.

3. Prove that the only compact subsets of the real line are closed and bounded.

4. Prove that for any two disjoint compact sets A and B in a metric space, there exists two disjoint open sets U and V such that A is contained in U and B is contained in V.

UNIT–II

5. Prove that any set contained between a connected set and its closure is also connected.

6. Let $\{A_n \mid n \in \mathbf{Z}^+\}$ be a countable collection of sets such that each A_n is connected and for each n, $A_n \cap A_{n+1}$ is non empty. Then prove that $\cup_n A_n$ is connected. Prove all the results that you use.

7. Prove that in a metric space, components of open sets are open if and only if every open set is a union of connected open sets.

8. Prove that every function of bounded variation is a difference of two bounded monotonic functions.

UNIT–III

9. State and prove the Cantors Intersection Theorem.

10. Prove that every metric space is a dense subspace of a complete metric space.

11. State and prove Banach's Contraction Principle.

12. State and prove a necessary and sufficient criteria for a metric space to be complete.

UNIT–IV

13. Prove that continuous image of a connected set is connected and continuous image of a compact set is compact.

14. Prove that a map $f : X \to Y$ between metric spaces is continuous at each point of X if and only if inverse image of each open subset of Y is open in X.

15. Let $f : A \to Y$ and $g : B \to Y$ be continuous maps, where $X = A \cup B$ such that A and B are disjoint and both A and B are either open in X or are both closed in X. Then prove that there is a continuous map $h : X \to Y$ such that $h \mid A = f$ and $h \mid B = g$.

16. Prove that every continuous function between metric spaces is uniformly continuous.

UNIT–V

17. State and prove a sufficient condition for the existence of the Riemann-Stieltjes Integral.

18. Prove that if f is Riemann Steiltjes integrable on [a, b] and $\int f dh = 0$ for every monotonic f then h is a constant function on [a, b].

19. State and prove the fundamental theorem of calculus.

20. State and prove the second mean value theorem for the Reimann-Stieltjes integral.